MATH-943 Convex Analysis

Credit hours: 3-0
Prerequisites: NONE

Course Objectives: Although the systematic study of convex sets started by the end of the 19th century, convexity only became an independent branch of mathematics by the middle of the 20th century. Convexity combines conceptual tools from geometry, analysis, linear algebra and topology, and plays a crucial role in number theory, optimization, inequality theory, combinatorial geometry and game theory. The course is focused on convex sets and convex functions, showing applications to optimality theory in convex programming and conjugacy theory.

Core Contents: Basic concepts of convex analysis, Topological properties of convex functions, Duality correspondence, Representation and inequalities and Bifunctions and generalized convex program.

Detailed Course Contents: Affine sets, convex sets and cones, the Algebra of convex sets, convex functions, functional operations, relative interiors of convex sets, closures of convex functions, some closeness criteria, continuity of convex functions, separation theorems, conjugates of convex functions, support functions, polers of convex sets and functions, dual operations, Caratheodory's theorem, extreme points and faces of convex sets, polyhedral convex sets and functions, some applications of polyhedral convexity, Helly's theorem and systems of inequalities, directional derivatives and sub gradients, constrained extremum problems, saddle functions and minimax theory.

Learning Outcomes: Students are expected to understand the fundamentals of convex analysis, Topological properties of convex functions, Duality correspondence, Representationand inequalities.

Text Book: R. Tyrrel Rockafeller, Convex Analysis, Princeton University press, 1970.

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35

End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	Part I Sec. 1,2	Affine sets, convex sets and cones
2	Part I Sec. 3,4	The Algebra of convex sets, convex functions
3	Part II Sec. 5, 6	Functional Operations, Relative interiors of convex sets
4	Part II Sec. 7, 8	Closures of convex functions, Recession cones andunboundedness
5	Part II Sec. 9, 10	Some closeness criteria, Continuity of convex functions
6	Part II Sec. 11, 12	Separation Theorems, Conjugates of convex functions
7	Part III Sec. 13, 14	Support function
8	Part III Sec. 14, 15	Polars of Convex sets, polars of convex functions
9	**Mid Semester Exam**	
10	Part III Sec. 16	Dual operations
11	Part IV Sec. 17, 18	Caratheodorey's Theorem, Extreme points and faces of convexsets
12	Part IV Sec. 19, 20	Polyhedral Convex sets and functions, Some applications ofPolyhedral convexity
13	Part IV Sec. 21, 22	Helly's Theorem and systems of inequalities, Linearinequalities
14	Part V Sec. 23,24	Directional derivatives and sub gradients, Differentialcontinuity and Monotonicity
15	Part VI Sec 27, 28	The minimum of a convex function, Ordinary convex programsand Lagrange multipliers
16	Part VI Sec 29, 30	Bifunctions and generalized convex program, Fenchel's dualitytheorem
17		Review
18	**End Semester Exam**	